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Anomalous transport in low-dimensional systems with correlated disorder

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Abstract
We review recent results on the anomalous transport in one-dimensional and quasi-one-dimensional systems with bulk and surface disorder. Principal attention is paid to the role of long-range correlations in random potentials for the bulk scattering and in corrugated profiles for the surface scattering. It is shown that with the proper type of correlations one can construct such a disorder that results in a selective transport with given properties. Of particular interest is the possibility to arrange windows of a complete transparency (or reflection) with dependence on the wave number of incoming classical waves or electrons.

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1. Introduction

The aim of this paper is two-fold. First, we review recent developments in the study of low-dimensional models with so-called *correlated disorder*. By this term we mean specific long-range correlations embedded in random potentials that lead to anomalous transport properties. Second, we observe new results obtained for one-dimensional (1D) and quasi-1D structures with corrugated surfaces resulting in *surface scattering*. In the latter problem, we also consider the case when surface profiles, although described by random functions, contain long-range correlations along the profiles.

Recently, the problem of anomalous transport in 1D systems with correlated disorder has attracted much attention. For the theory of disordered systems, the fundamental significance of this problem is due to exciting results that revise a commonly accepted belief that any randomness in long 1D structures results in Anderson localization. From the experimental viewpoint, many of the results may have a strong impact for the creation of a new class of

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electron nanodevices, optic fibres, acoustic and electromagnetic waveguides with selective transport properties.

As was shown [1–3], specific long-range correlations in random potentials can give rise to the appearance of the *mobility edges* for 1D finite structures. In particular, an interval of electron energy (or wave frequency) arises on one side of the mobility edge where the eigenstates turn out to be extended, in contrast to the other side where the eigenstates remain exponentially localized. The position and the width of the windows of transparency can be controlled by the form of the binary correlator of a scattering potential. A quite simple method was used for constructing random potentials that result in any predefined energy/frequency window of a perfect transparency.

In order to avoid any confusion, we have to stress here that by the mobility edge we mean the situation when on one side of this 'edge' the inverse localization length L_{loc}^{-1} is determined by the first order of perturbation theory and, therefore, is proportional to the variance V_0^2 of a weak disordered potential. On the other side, L_{loc}^{-1} abruptly decreases to the value determined by the next-order terms which are proportional to the squared variance, V_0^4, or higher. Thus, there is no conflict with the well-known result [4] that in *infinite* samples the localization length is always finite in a low-dimensional geometry with random potentials. As is also known, for specific values of energy the localization length can diverge; however, the measure of such states is zero.

We would like to stress that the effect we discuss here is of great interest in view of different applications. Indeed, in practice the disordered samples are always finite, therefore, it is worthwhile to speak about 'the mobility edge in the first-order approximation'. This means that having a fixed size L of a sample, one can arrange the situation (by a proper choice of a weak disorder with long-range correlations), for which on one side of the 'mobility edge' in the energy space the localization length is much smaller in comparison with the sample size, $L_{\text{loc}} \sim V_0^{-2} \ll L$; however, on the other side of this border it diverges in the first-order approximation. Thus, in order to observe a sharp transition from completely transparent states to strongly localized ones, one should assume that the localization length determined by the next terms of the perturbation theory is always much larger than the sample size, $L_{\text{loc}}^{(2)} \sim V_0^{-4} \gg L$. As was confirmed experimentally this transition is feasible, and may be practically considered as a kind of 'mobility edge' in 1D or quasi-1D finite samples. Note also that due to a correlated disorder the localization length can be strongly reduced to a small value in narrow energy windows, a fact which is highly non-trivial (see below and [5]).

The predictions of the theory have been verified experimentally [6, 7] by studying transport properties of a single-mode electromagnetic waveguide with point-like scatters. The latter were intentionally inserted into the waveguide in order to provide random potential with a given binary correlator. In the experiment, the disorder was created by the array of 500 screws with random heights obtained numerically in accordance with the analytical expressions. The single-mode regime was provided within the frequency range $\nu = 7.5$–15 GHz. The agreement between experimental and numerical data was found to be unexpectedly good. In particular, the predicted windows of a complete reflection alternated by those of a good transparency were clearly observed, in spite of many experimental imperfections.

Main results on the anomalous transport and mobility edges are based on a new transfer matrix approach [8–10] that was found to be quite effective in the study of localization properties of eigenstates in 1D geometry. This approach established a direct relevance of Anderson localization to the parametric instability of classical linear oscillators with white and coloured noise [11]. With this approach, the role of long-range correlations has been substantially studied in 1D discrete tight-binding models of the Anderson and Kronig–Penney types [1, 2, 9].

A further development of the theory is due to its application to the surface scattering of electromagnetic waves (or electrons) in quasi-1D guiding systems. The problem of wave propagation (both classical and quantum) through such systems with corrugated surfaces has a quite long history and till now remains a hot topic in the literature. This problem naturally arises in the analysis of spectral and transport properties of optics fibres, acoustic and radio waveguides, remote sensing, shallow water waves, multilayered systems and photonic lattices, etc [12, 13]. Also, similar problems arise when describing the spectral and transport properties of quantum quasi-particles in thin metal films and semiconductor nanostructures, such as nanowires and strips, superlattices and quantum-well-systems [14–20].

As is well established, scattering from corrugated surfaces results in classical and quantum diffusive transport [21], as well as in the effects of strong electron/wave localization [22–29]. Correspondingly, the eigenstates of periodic systems with corrugated surfaces turn out to have a chaotic structure [30]. Recent numerical studies of quasi-1D surface-disordered systems [28, 29] have revealed a principal difference from those known in the standard models with *bulk* random potentials [19]. Specifically, it was found that transport properties of quasi-1D waveguides with rough surfaces essentially depend on many characteristic lengths, in contrast to the bulk scattering where the *one-parameter scaling* occurs. According to this remarkable scaling, all scattering properties of finite samples with bulk disorder are determined by the ratio of the localization length found for infinite systems to the size of the samples. The situation for the surface scattering in quasi-1D structures is principally different due to a non-isotropic character of surface scattering in the 'channel space'. In particular, the transmission coefficient for the surface scattering substantially decreases with an increase of the angle of incoming waves.

One should stress that main results in the theory of surface scattering were obtained for random surfaces with fast-decaying correlations along the structures. Therefore, it is of great importance to explore the role of specific long-range correlations in surface profiles, having in mind the results found for 1D systems with correlated disorder. Apart from the theoretical interest, this problem can find many applications since existing experimental techniques allow for the construction of systems with sophisticated surfaces and bulk-scattering potentials resulting in anomalous transport properties [31–33].

For single-mode waveguides with the surface scattering the problem turns out to be equivalent to the 1D bulk scattering. For this reason, the methods and results obtained for the latter case can be directly applied for the waveguides [34–36]. Specifically it was shown, both analytically and by direct numerical simulations, that single-mode waveguides with a desired selective transport can be fabricated by a proper construction of long-range-correlated random surfaces.

A much more tricky situation arises in multi-mode waveguides [36–38]. As was shown, in this case the long-range correlations, on the one hand, give rise to a suppression of the interaction between different propagating waveguide modes. On the other hand, the same correlations can provide a perfect transparency of each independent channel, similar to what happens in the 1D geometry. The number of independent transparent modes is governed by the correlation length and can be as large as the total number of propagating modes. Therefore, transmission through a waveguide can be significantly enhanced in comparison with the case of uncorrelated surface roughness. The important point is that for multi-mode surface scattering there are many scattering lengths and the one-parameter scaling is not valid. As a result, the transport through different channels can be separated by a proper choice of long-range correlations. Thus, the intra-mode transitions from ballistic to diffusive or localized regime are expected to be sharp enough in order to observe the corresponding mobility edges experimentally.

In order to understand the impact of long-range correlations in quasi-1D geometry, we have also discussed the anomalous transport and mobility edges in waveguides with another kind of disorder, the so-called *stratified* or *layered disorder* [39, 40]. In spite of the apparent simplicity of this model, the results turn out to be quite instructive. This model describes the scattering of classical waves or electrons through quasi-1D structures with a disorder that depends on the longitudinal coordinate only. Therefore, it absorbs both the properties of 1D correlated structures and those that are specific for quasi-1D systems with surface scattering. On the one hand, the transport through any of open channels is independent from the others. On the other hand, since the localization length in each channel strongly depends on its number, the total transmittance is a complicated combination of the partial 1D transmittances. As a result, the effect of long-range correlations in such a model turns out to be highly non-trivial, leading to quite unexpected phenomena. In particular, it is shown that with a proper choice of the binary correlator one can arrange a situation where some of the channels are completely transparent and the others are closed. This interplay between localized and transparent channels gives rise to the effect of a *non-monotonic* stepwise dependence of the transmittance on the wave number. The results may find practical applications for fabrication of electromagnetic/acoustic waveguides, optic fibres and electron nano-conductors that reveal non-conventional selective transport.

The structure of this paper is as follows. In section 2, the 1D model with a random potential is introduced, for which the main expressions for the transmission coefficient of finite samples are given. With the use of these expressions, we discuss the notion of single parameter scaling, as well as the role of long-range correlations for the anomalous transport. In section 3, the quasi-1D model with a stratified disorder is considered in detail. First, it is shown that in this model a hierarchy of characteristic scattering lengths arises resulting in a coexistence of ballistic and localized regimes. We shown how this effect gives rise to unexpected transport properties for the potentials with specific long-range correlations. In order to demonstrate these effects, a simple example is analysed manifesting a highly non-trivial dependence of the transmittance on the energy of incident waves.

Section 4 is devoted to the scattering through guiding systems with corrugated surfaces. In the particular case of single-mode waveguides, the analysis of surface scattering is shown to be analogous to the bulk scattering in 1D structures. Using this analogy, we show how to construct long-range correlations along the scattering profiles, in order to observe the windows of a complete transparency alternated by the regions of a complete reflection. The analytical results are demonstrated by a direct numerical computation of the localization length for the reconstructed potential with a given two-point correlator. Finally, in the same section the study of a multi-mode waveguide with correlated surface profiles is performed, taking as an example the potential with the step function for the roughness power spectrum of a disorder. It is shown that these kinds of correlations may lead to a fractional dependence of the transmission coefficient on the normalized wave vector of incoming waves.

2. One-dimensional transport in random media

Let us start with the stationary one-dimensional Schrödinger (or wave) equation

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} + k^2 - V(x)\right]\Psi(x) = 0 \tag{2.1}$$

with the wave number k equal to ω/c for a classical scalar wave of frequency ω or to the Fermi wave number for electrons within the isotropic Fermi-liquid model. The static potential $V(x)$ for electromagnetic waves is given by $V(x) = k^2\delta\epsilon(x)$ with the dielectric constant

$\epsilon(x) = 1 - \delta\epsilon(x)$. For electrons, we use the units in which $2m/\hbar^2 = 1$. Below we assume that the statistically homogeneous, isotropic random function $V(x)$ is determined by standard properties,

$$\langle V(x)\rangle = 0, \qquad \langle V^2(x)\rangle = V_0^2, \qquad \langle V(x)V(x')\rangle = V_0^2\mathcal{W}(x-x'). \tag{2.2}$$

Here, the angular brackets $\langle\cdots\rangle$ stand for the statistical average over different realizations of the random potential $V(x)$ and the variance is denoted by V_0^2. The binary (two-point) correlator $\mathcal{W}(x-x')$ is normalized to $\mathcal{W}(0) = 1$ at $x = x'$ and is assumed to decrease with increasing the distance $|x - x'|$.

In what follows we consider weak over-barrier scattering, $V_0 \ll k^2$, for which appropriate perturbative approaches can be used. In this case, all transport properties are entirely determined by the *randomness power spectrum* $W(k_x)$,

$$\mathcal{W}(x) = \int_{-\infty}^{\infty} \frac{\mathrm{d}k_x}{2\pi} \exp(\mathrm{i}k_x x) W(k_x), \tag{2.3a}$$

$$W(k_x) = \int_{-\infty}^{\infty} \mathrm{d}x \exp(-\mathrm{i}k_x x)\mathcal{W}(x). \tag{2.3b}$$

Since the correlator $\mathcal{W}(x)$ is a real and even function of the coordinate x, its Fourier transform (2.3b) is even, real and non-negative function of the wave number k_x. The condition $\mathcal{W}(0) = 1$ results in the following normalization for $W(k_x)$:

$$\int_{-\infty}^{\infty} \frac{\mathrm{d}k_x}{2\pi} W(k_x) = 1. \tag{2.4}$$

We shall characterize the transport properties of finite open systems by the wave *transmittance* or, the same, by the electron *dimensionless conductance* T. In the case of the electron transport, T is defined as the conductance G in units of $\mathrm{e}^2/\pi\hbar$,

$$T = \frac{G}{\mathrm{e}^2/\pi\hbar}. \tag{2.5}$$

The transmittance T for our model (2.1)–(2.2) can be found with the use of well-developed methods, such as, e.g., the perturbative diagrammatic technique of Berezinski [15], the invariant imbedding method [41] or the two-scale approach [26, 42]. All the methods allow us to take adequately into account the effects of coherent multiple electron/wave scattering giving rise to Anderson localization.

2.1. One-parameter scaling

Main theoretical result is that the *average transmittance* $\langle T\rangle$ as well as all moments $\langle T^s\rangle$ are described by the universal function,

$$\begin{aligned}\langle T^s(L/L_{\mathrm{loc}})\rangle = {}& \sqrt{\frac{2}{\pi}}\left(\frac{L_{\mathrm{loc}}}{L}\right)^{3/2} \exp\left(-\frac{L}{2L_{\mathrm{loc}}}\right)\\ &\times \int_0^{\infty} \frac{z\,\mathrm{d}z}{\cosh^{2s-1} z} \exp\left(-z^2\frac{L_{\mathrm{loc}}}{2L}\right)\int_0^z \mathrm{d}y \cosh^{2(s-1)} y,\\ & s = 0, \pm 1, \pm 2, \ldots . \end{aligned} \tag{2.6}$$

This function depends solely on the scaling parameter L/L_{loc} which is the ratio between the length L and the so-called *localization length* L_{loc} [16]. The quantity $L_{\mathrm{loc}}/2$ is, in fact, the *backscattering length* in an infinite sample with the same disorder. The inverse value L_{loc}^{-1}

can be determined via the Lyapunov exponent of the corresponding averaged wavefunction $\langle\Psi(x)\rangle$ within the transfer matrix approach. Such a dependence of the transport properties on the ratio L/L_{loc} manifests a principal concept of *one-parameter scaling* that constitutes the phenomenon of 1D Anderson localization.

From equation (2.6) one can find relatively easily the low moments of the distribution of the transmittance T. Specifically, at $s = -1$ one obtains the average dimensionless resistance $\langle T^{-1}(L/L_{\text{loc}})\rangle$. It is given by the surprisingly simple formula,

$$\langle T^{-1}(L/L_{\text{loc}})\rangle = \frac{1}{2}\left[1 + \exp\left(\frac{4L}{L_{\text{loc}}}\right)\right], \tag{2.7}$$

that demonstrates an exponential increase of the resistance with an increase of the sample length L or with a decrease of the localization length L_{loc}.

The average transmittance (dimensionless conductance) $\langle T(L/L_{\text{loc}})\rangle$ is described by equation (2.6) with $s = 1$, leading to the following expression:

$$\langle T(L/L_{\text{loc}})\rangle = \sqrt{\frac{2}{\pi}}\left(\frac{L_{\text{loc}}}{L}\right)^{3/2} \exp\left(-\frac{L}{2L_{\text{loc}}}\right)\int_0^\infty \frac{z^2\,\mathrm{d}z}{\cosh z}\exp\left(-z^2\frac{L_{\text{loc}}}{2L}\right). \tag{2.8}$$

As for the second moments, $s = 2$ and $s = -2$, they define the variance of the conductance and resistance, respectively. One can see that the average resistance does not coincide with the inverse value of the average conductance, $\langle T^{-1}\rangle \neq \langle T\rangle^{-1}$. Also, from equation (2.6), one can find that at $L_{\text{loc}} \lesssim L$, the variance of the resistance and conductance is of the order of their squared average values themselves. This fact means that in this case both the conductance and resistance are not self-averaged quantities. Hence, by changing the length L of the sample or the disorder itself for the same L, one should expect large fluctuations of the conductance and resistance. This type of fluctuations are known as the *mesoscopic fluctuations* which are characteristic of strong quantum effects on the macroscopic scale.

In order to properly characterize the transport properties of samples with exponentially small conductance, one should refer to the self-averaged logarithm of the conductance,

$$\langle \ln T(L/L_{\text{loc}})\rangle = -2L/L_{\text{loc}}. \tag{2.9}$$

This result shows an exponential decrease of the transmittance for the so-called representative (non-resonant) realizations of the random potential [16]. Note that this expression can serve as the definition of the localization length L_{loc} itself, which is complementary to the definition of L_{loc} through the Lyapunov exponent in the transfer matrix method.

In accordance with the one-parameter scaling concept, there are only *two transport regimes* in the 1D-disordered structures, the regimes of the ballistic and localized transport.

(i) The *ballistic transport* occurs if the localization length L_{loc} is much larger than the sample length L. In this case, the samples are practically fully transparent,

$$\langle T(L/L_{\text{loc}})\rangle \approx 1 - 2L/L_{\text{loc}} \qquad \text{for} \quad L_{\text{loc}} \gg L. \tag{2.10}$$

(ii) Otherwise, the 1D-disordered structures exhibit the *localized transport* when the localization length L_{loc} is small enough in comparison with the sample length L. In this case, the average transmittance is exponentially small,

$$\langle T(L/L_{\text{loc}})\rangle \approx \frac{\pi^3}{4\sqrt{2\pi}}(L/L_{\text{loc}})^{-3/2}\exp(-L/2L_{\text{loc}}) \qquad \text{for} \quad L_{\text{loc}} \ll L. \tag{2.11}$$

As one can see, in the localization regime a 1D-disordered system almost perfectly (with an exponential accuracy) reflects the electrons (or waves) due to a *strong localization* of all eigenstates.

2.2. Binary correlator

The expressions (2.6)–(2.11) are universal and applicable for any one-dimensional system with a *weak static disorder*. As one can see, in order to describe transport properties of finite sample, one needs to know the localization length L_{loc} which is a characteristic of Anderson localization occurring in infinite samples. According to different approaches [15, 16, 41, 42], the inverse localization length for any kind of weak disorder is determined by the $2k$-harmonic in the randomness power spectrum $W(k_x)$ of the scattering potential $V(x)$,

$$L_{\mathrm{loc}}^{-1}(k) = \frac{V_0^2}{8k^2} W(2k). \tag{2.12}$$

This canonical expression indicates that all features of the electron/wave transmission through the 1D-disordered media depend on the two-point correlations in the random scattering potential. Therefore, if the power spectrum $W(2k)$ is very small or vanishes within some interval of the wave number k, the localization length L_{loc} turns out to be very large ($L_{\mathrm{loc}} \gg L$) or even diverges. Evidently, the localization effects can be neglected in this case and the conductor even of a large length should be almost fully transparent. This means that, in principle, by a proper choice of the disorder one can design the disordered structures with selective (*anomalous*) ballistic transport within a prescribed range of k.

Before we start with a practical implementation of the above expression (2.12) for the inverse localization length, it is worthwhile to clear up some points. First, one should stress that this result, obtained by different methods, is an asymptotic one. This means that the higher terms are non-controlled; however, they can be neglected in the limit $V_0^2 \to 0$. Second, the main assumptions used in the derivation of this expression are based on the validity of averaging over different realization over disorder. In particular, a possibility of introducing fast and slow variables is assumed that allows us to perform such an averaging. As is shown (see, for instance, [42]), the condition justifying these approaches is that two 'slow' lengths, localization length L_{loc} and sample size L, are much larger than two 'fast' lengths, wavelength k^{-1} and the correlation length R_c that determines the maximal value of the power spectrum $W(2k)$. One should stress that for any fixed values of k and R_c, this condition can be always fulfilled due to an asymptotic character of the expression (2.12) (or, the same, due to a non-restricted small value of V_0^2).

Thus, the important practical problem arises of how to construct such a random potential $V(x)$ from a prescribed power spectrum $W(k_x)$. This problem can be solved by a direct generalization of the method developed in [1, 2] for discrete 1D models with random potentials. In our case of the *continuous* random potential $V(x)$, the solution of the problem is as follows. First, starting from a given form of the power spectrum $W(k_x)$, one should obtain the function $\beta(x)$ whose Fourier transform is $W^{1/2}(k_x)$,

$$\beta(x) = \int_{-\infty}^{\infty} \frac{\mathrm{d}k_x}{2\pi} \exp(\mathrm{i}k_x x) W^{1/2}(k_x). \tag{2.13}$$

After this, the random profile of the scattering potential $V(x)$ can be generated as a convolution of a delta-correlated random process $Z(x)$ with the function $\beta(x)$,

$$V(x) = V_0 \int_{-\infty}^{\infty} \mathrm{d}x'\, Z(x - x')\beta(x'). \tag{2.14}$$

Here, the 'white noise' $Z(x)$ is determined by the standard properties,

$$\langle Z(x)\rangle = 0, \qquad \langle Z(x)Z(x')\rangle = \delta(x - x'), \tag{2.15}$$

and can be easily created with the use of random number generators.

The above expressions allow us to solve the inverse scattering problem of constructing random potentials from their power spectrum. Note that this construction is possible in the case of a weak disorder only. That is why only the binary correlator is involved in the reconstruction of $V(x)$ while the higher correlators do not contribute. Note also that the potential obtained by the proposed method is not unique. Indeed, there is an infinite set of delta-correlated random processes $Z(x)$. Therefore, substituting them into the definition (2.14), we come to an infinite number of random potential profiles with a given power spectrum $W(k_x)$.

The important point in this approach is that one can arrange a very *sharp transition* between ballistic and localized regimes of electron/wave transport at any prescribed one or more points on k-axis. It is clear that in this case the power spectrum $W(k_x)$ must abruptly vanish at the prescribed points. This means that the binary correlator $\mathcal{W}(x-x')$ has to be a slowly decaying function of the distance $x-x'$. In other words, random scattering potentials $V(x)$ that give rise to the combination of ballistic and localized transport windows should be of specific form with long-range correlations along the structure. Because of an abrupt character, the transition points can be regarded as *mobility edges*.

As is pointed out above, a statistical treatment is meaningful if the scale of decrease $R_c \sim k_c^{-1}$ of the correlator $\mathcal{W}(x)$ is much less than the sample length L and the localization length. In this connection, one should stress that long-range correlations we speak about do not assume large values for R_c. Indeed, the simplest correlator, $\sin k_c x/k_c x$, has finite scale k_c^{-1} of its decrease, that can be quite small, see examples below. At the same time, an effective width of the mobility edge turns out to be very small since it is determined by the product, $(L_{\mathrm{loc}}k_c)^{-1}$, not by k_c^{-1} (for details, see, e.g., [42]). One can say that the sharpness of the transition is defined by the form of the pair correlator rather than by the value of its correlation length.

2.3. Mobility edges

Let us now demonstrate the suggested approach of constructing the potentials giving rise in the mobility edges. For this, we consider the power spectrum of the following rectangular form:

$$\begin{aligned} W(k_x) &= \frac{\pi}{2(k_+-k_-)}\Theta(2k_+-|k_x|)\Theta(|k_x|-2k_-) \\ &= \frac{\pi}{2(k_+-k_-)}\left[\Theta(2k_+-|k_x|)-\Theta(2k_--|k_x|)\right], \qquad k_+>k_->0. \end{aligned} \tag{2.16}$$

Here, $\Theta(x)$ is the Heaviside unit-step function, $\Theta(x<0)=0$ and $\Theta(x>0)=1$. The characteristic wave numbers $k_\pm$ are the correlation parameters to be specified. Note that such a power spectrum was recently employed to create a special rough surface for the experimental study of an enhanced backscattering [31]. Moreover, it was used in the theoretical analysis of light scattering from amplifying media [43], as well as of the localization of surface plasmon polaritons on random surfaces [44].

According to equation (2.3*a*) the binary correlator $\mathcal{W}(x)$ corresponding to the expression (2.16) has the following form:

$$\mathcal{W}(x) = \frac{\sin(2k_+x)-\sin(2k_-x)}{2(k_+-k_-)x}. \tag{2.17}$$

In equations (2.16) and (2.17), the factor $1/2(k_+-k_-)$ stands to provide the normalization requirement (2.4) or the same, $\mathcal{W}(0)=1$.

Following the suggested recipe (2.13)–(2.15), one can construct the scattering potential $V(x)$ with the above correlation properties,

$$V(x) = \frac{V_0}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathrm{d}x'\, Z(x-x') \frac{\sin(2k_+x') - \sin(2k_-x')}{(k_+ - k_-)^{1/2} x'}. \tag{2.18}$$

One should stress that the potential (2.18) with the binary correlator (2.17) and power spectrum (2.16) is substantially different from the widely used delta-correlated noise or random processes with fast-decaying Gaussian correlations. Here, the potential (2.18) is specified by two variation parameters, $(2k_+)^{-1}$ and $(2k_-)^{-1}$, and has long tails in the expression for the two-point correlator (2.17). The existence of such tails manifests the long-range correlations in the potential, originated from the stepwise discontinuities at the points $k_x = \pm 2k_\pm$ in its power spectrum (2.16). Note that, in fact, these two parameters, $(2k_+)^{-1}$ and $(2k_-)^{-1}$, are nothing but two characteristic scales of the two-point correlator.

From equations (2.12) and (2.16), one can find the inverse localization length

$$\frac{1}{L_{\mathrm{loc}}(k)} = \frac{\pi V_0^2}{16(k_+ - k_-)} \frac{\Theta(k_+ - k)\Theta(k - k_-)}{k^2}. \tag{2.19}$$

As one can see, there are two mobility edges at the points $k = k_-$ and $k = k_+$. The localization length $L_{\mathrm{loc}}(k)$ diverges below the first point, $k = k_-$, and above the second one, $k = k_+$. Between these points, for $k_- < k < k_+$, the localization length (2.19) has finite value and smoothly increases with an increase of wave number k.

Let us now choose the parameters for which the regime of a strong localization occurs at the upper transition point $k = k_+$. This automatically provides the strong localization within the whole interval $k_- < k < k_+$. The condition of such situation reads

$$\frac{L}{L_{\mathrm{loc}}(k_+)} = \frac{\pi V_0^2 L}{16k_+^2(k_+ - k_-)} \gg 1. \tag{2.20}$$

As a result, there are two regions of transparency for the samples of finite length L with the chosen kind of random potentials. Between these regions the average transmittance $\langle T \rangle$ is exponentially small according to the expression (2.11). Due to this fact, the system exhibits the localized transport within the interval $k_- < k < k_+$ and the ballistic regime with $\langle T \rangle = 1$ outside this interval. In the experiment one can observe that with an increase of the wave number k, the perfect transparency below $k = k_-$ abruptly alternates with a complete reflection and recovers at $k = k_+$. From equations (2.19) and (2.20), one can see that the smaller value $k_+ - k_-$ of the reflecting region the smaller the length $L_{\mathrm{loc}}(k)$ and, consequently, the stronger is the localization within this region. This remarkable fact may find important applications in creating a new class of random narrow-band filters or reflectors.

3. Quasi-one-dimensional stratified structures

Now we extend our study of the correlated disorder to the quasi-one-dimensional (quasi-1D) systems. To this end, we consider a plane waveguide (or conducting electron wire) of width d, stretched along the x-axis. The z-axis is directed in transverse to the waveguide direction so that one (lower) edge of the waveguide is $z = 0$ and the other (upper) edge is $z = d$. The waveguide is assumed to have a *stratified* disorder inside the finite region $|x| < L/2$ of length L, which depends on x only. Thus, the scattering is confined within a region of the $\{x, z\}$-plane defined by

$$-L/2 < x < L/2, \qquad 0 \leqslant z \leqslant d. \tag{3.1}$$

As a physically plausible model for stratified disorder, we shall employ the statistically homogeneous and isotropic random potential $V(x)$ with the correlation properties (2.2)–(2.4). One should note that in this case the scale of a decrease of the binary correlator $\mathcal{W}(x)$ is of the order of the characteristic scale of stratification.

The wavefunction obeys the two-dimensional Schrödinger equation

$$\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2 - V(x)\right]\Psi(x,z) = 0. \tag{3.2}$$

This equation is complemented by zero Dirichlet boundary conditions on both walls of the waveguide, $\Psi(x, z = 0) = \Psi(x, z = d) = 0$.

Since the scattering potential $V(x)$ depends only on the lengthwise coordinate x, the waveguide eigenfunction $\Psi(x,z)$ is naturally presented in the canonical form of the normal waveguide modes,

$$\Psi(x,z) = \left(\frac{2}{d}\right)^{1/2} \sin\left(\frac{\pi n z}{d}\right)\psi_n(x). \tag{3.3}$$

Its longitudinal mode-component $\psi_n(x)$ is governed by the quasi-one-dimensional wave equation that directly follows from equations (3.2) and (3.3),

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} + k_n^2 - V(x)\right]\psi_n(x) = 0. \tag{3.4}$$

Here, the wave number k_n is a quantum value of the lengthwise wave number k_x for nth waveguide mode,

$$k_n = \sqrt{k^2 - (\pi n/d)^2}. \tag{3.5}$$

Evidently, the transport properties are contributed only by those waveguide modes that can propagate along the structure (along x-axis), i.e. have real value of k_n. These propagating modes occupy the so-called conducting channels. As follows from equation (3.5), the total number N_d of propagating modes is equal to the integer part $[\cdots]$ of the ratio kd/π,

$$N_d = [kd/\pi]. \tag{3.6}$$

The waveguide modes with the indices $n > N_d$ corresponding to imaginary values of k_n and not contributing to the electron/wave transmission are called the evanescent modes.

From the wave equation (3.4), one can see that since the stratified disorder does not depend on z, there is no coupling between the propagating modes. Therefore, the stratified wire represents a set of 1D non-interacting conducting channels. That is why, in full accordance with the Landauer's concept [18], the total transmittance T of the stratified waveguide can be expressed as a sum of independent partial transmittances T_n corresponding to every nth propagating mode,

$$T(L) = \sum_{n=1}^{N_d} T_n(L). \tag{3.7}$$

In such a way we have reduced the transport problem for the quasi-1D-disordered system to the consideration of the transport through a number of 1D structures with the random potential $V(x)$. The scattering inside each of these structures is described by equation (3.4) and is entirely consistent with the phenomenon of the 1D Anderson localization. Specifically, the average mode-transmittance $\langle T_n\rangle$ is described by the universal expressions (2.6)–(2.11). The *mode localization length* $L_{\mathrm{loc}}(k_n)$, associated with a specific nth channel, is determined by equation (2.12) in which the total wave number k should be replaced by the lengthwise wave number k_n,

$$L_{\text{loc}}^{-1}(k_n) = \frac{V_0^2}{8k_n^2} W(2k_n). \tag{3.8}$$

Thus, the partial transport in each nth conducting channel obeys the concept of the *one-parameter scaling*. In particular, for $L_{\text{loc}}(k_n)/L \gg 1$, the average transmittance $\langle T_n \rangle$ exhibits the ballistic behaviour and the corresponding nth normal mode is practically transparent, see equation (2.10). In contrast, the transmittance $\langle T_n \rangle$ is exponentially small in line with equation (2.11), when the mode localization length is much less than the length of the waveguide, $L_{\text{loc}}(k_n)/L \ll 1$. This implies strong electron/wave localization in the nth channel.

3.1. Coexistence of ballistic and localized transport

The main feature of the mode localization length $L_{\text{loc}}(k_n)$ is its rather strong dependence on the channel index n. One can see from equation (3.8) that the larger n is, the smaller the mode localization length $L_{\text{loc}}(k_n)$ and, consequently, the stronger the coherent scattering within this mode. This strong dependence is due to the squared wave number k_n in the denominator of equation (3.8). Evidently, with an increase of the mode index n the value of k_n decreases. An additional dependence appears because of the stratification power spectrum $W(2k_n)$. Since the binary correlator $\mathcal{W}(x)$ of the random stratification is a decreasing function of $|x|$, the numerator $W(2k_n)$ increases with n (note that it is a constant for the delta-correlated stratification only). Therefore, both the numerator and denominator contribute in the same direction for the dependence of $L_{\text{loc}}(k_n)$ on n. As a result, we arrive at the hierarchy of mode localization lengths,

$$L_{\text{loc}}\left(k_{N_d}\right) < L_{\text{loc}}\left(k_{N_d-1}\right) < \cdots < L_{\text{loc}}(k_2) < L_{\text{loc}}(k_1). \tag{3.9}$$

The smallest mode localization length $L_{\text{loc}}\left(k_{N_d}\right)$ belongs to the highest (last) channel with the mode index $n = N_d$, while the largest mode localization length $L_{\text{loc}}(k_1)$ corresponds to the lowest (first) channel with $n = 1$. Note that a similar hierarchy was also found in [28, 29] for the attenuation lengths in the model of quasi-1D waveguides with rough surfaces.

Thus, a remarkable phenomenon arises. On the one hand, the concept of the one-parameter scaling holds for any of the N_d conducting channels whose partial transport is characterized solely by the ratio $L/L_{\text{loc}}(k_n)$. On the other hand, this concept turns out to be broken for the total waveguide transport. Indeed, due to the revealed hierarchy (3.9) of the mode localization lengths $L_{\text{loc}}(k_n)$, the total average transmittance (3.7) depends on the whole set of scaling parameters $L/L_{\text{loc}}(k_n)$. This fact is in contrast with quasi-1D bulk-disordered models, for which all transport properties are shown to be described by one parameter only.

The interplay between the hierarchy of the mode localization lengths $L_{\text{loc}}(k_n)$ on the one hand and the one-parameter scaling for every partial mode-transmittance $\langle T_n \rangle$ on the other hand gives rise to a new phenomenon of the intermediate coexistence regime, in addition to the regimes of ballistic or localized transport known in the 1D geometry.

(i) *Ballistic transport.* If the *smallest* mode localization length $L_{\text{loc}}\left(k_{N_d}\right)$ is much *larger* than the scattering region of size L, all conducting channels are open. They have almost unit partial transmittance, $T_n(L) \approx 1$, hence, the stratified waveguide is fully transparent. In this case, the total average transmittance (3.7) is equal to the total number of the propagating modes,

$$\langle T(L) \rangle \approx N_d \qquad \text{for} \quad L_{\text{loc}}\left(k_{N_d}\right) \gg L. \tag{3.10}$$

(ii) *Localized transport*. In contrast, if the *largest* of the mode localization lengths $L_{\mathrm{loc}}(k_1)$ turns out to be much *smaller* than the waveguide length L, all the propagating modes are strongly localized and the waveguide is non-transparent. Its total average transmittance, in accordance with equations (3.7) and (2.11), is exponentially small,

$$\langle T(L)\rangle \approx \frac{\pi^3}{4\sqrt{2\pi}}\,[L/L_{\mathrm{loc}}(k_1)]^{-3/2}\exp\left[-L/2L_{\mathrm{loc}}(k_1)\right] \qquad \text{for} \quad L_{\mathrm{loc}}(k_1) \ll L. \quad (3.11)$$

(iii) *Coexistence transport*. The intermediate situation arises when the *smallest* localization length $L_{\mathrm{loc}}(k_{N_d})$ of the last (highest) N_d th mode is *smaller*, while the *largest* localization length $L_{\mathrm{loc}}(k_1)$ of the first (lowest) mode is *larger* than the waveguide length L,

$$L_{\mathrm{loc}}(k_{N_d}) \ll L \ll L_{\mathrm{loc}}(k_1). \quad (3.12)$$

In this case, an interesting phenomenon of the coexistence of ballistic and localized transport occurs. Namely, while the *lower* modes are in the ballistic regime, the *higher* modes display the strongly localized behaviour.

These transport regimes can be observed experimentally when, e.g., the stratification is either the random delta-correlated process of white-noise type with a constant power spectrum $V_0^2 W(k_x) = W_0 = \text{const.}$ or has Gaussian correlations. As a demonstration, let us consider the stratified waveguide with a large number of conducting channels,

$$N_d = [kd/\pi] \approx kd/\pi \gg 1, \quad (3.13)$$

and with a random potential $V(x)$ having the widely used Gaussian correlator,

$$\mathcal{W}(x) = \exp\left(-k_0^2 x^2\right), \quad (3.14a)$$

$$W(k_x) = \sqrt{\pi}\,k_0^{-1}\exp\left(-k_x^2/4k_0^2\right). \quad (3.14b)$$

It is convenient to introduce two parameters

$$\alpha = \frac{L}{L_{\mathrm{loc}}^{w}(k_1)} = \frac{W_0 L}{8k^2}, \qquad \delta = \frac{L_{\mathrm{loc}}^{w}(k_{N_d})}{L_{\mathrm{loc}}^{w}(k_1)} = \frac{2\{kd/\pi\}}{(kd/\pi)} \ll 1, \quad (3.15)$$

where $L_{\mathrm{loc}}^{w}(k_1)$ and $L_{\mathrm{loc}}^{w}(k_{N_d})$ refer, respectively, to the largest and smallest mode localization lengths in the limit case of the white-noise potential ($k_0 \to \infty$, $\sqrt{\pi}\,V_0^2 k_0^{-1} = W_0 = \text{const}$). Here, $\{kd/\pi\}$ is the fractional part of the mode parameter kd/π.

One can find that for the Gaussian correlations (3.14*a*), (3.14*b*), all propagating modes are strongly localized when

$$\exp\left(k^2/k_0^2\right) \ll \alpha. \quad (3.16)$$

Note that this inequality is stronger than that valid for the white-noise case, $\alpha \gg 1$.

The intermediate situation with the coexistence transport occurs when

$$\delta\exp\left(\delta k^2/k_0^2\right) \ll \alpha \ll \exp\left(k^2/k_0^2\right). \quad (3.17)$$

Therefore, the longer the range k_0^{-1} of the correlated disorder, the simpler the conditions (3.17) of the *coexistence* of ballistic and localized transport.

Finally, the waveguide is almost perfectly transparent in the case when

$$\alpha \ll \delta\exp\left(\delta k^2/k_0^2\right). \quad (3.18)$$

From this brief analysis one can conclude that the localization is easily achieved for the white-noise stratification when the condition $\alpha \gg 1$, weaker than (3.16), should be met. However, at any given value of $\alpha \gg 1$ (fixed values of the waveguide length L, wave number k and the disorder strength W_0), the intermediate or ballistic transport can be realized by a proper choice of the stratified correlated disorder, i.e. with a proper choice of the correlation scale k_0^{-1}. From equation (3.17), one can also understand that for the standard white-noise case, the coexistence of ballistic and localized modes can be observed under the conditions $\delta \ll \alpha \ll 1$. If $\alpha \ll \delta$, the quasi-1D structure with the white-noise stratification displays the ballistic transport.

3.2. Correlated stratification and stepwise non-monotonic transmittance

From the above analysis it becomes clear that in quasi-1D stratified guiding structures with delta-correlated or Gaussian correlations, the crossover from the ballistic to localized transport is realized through the successive localization of highest propagating modes. Otherwise, if we start from the localized transport regime, the crossover to the ballistic transport is realized via the successive opening (delocalization) of the lowest conducting channels.

A fundamentally different situation arises when random stratified media have specific long-range correlations. To show this, we would like to note that the mode localization length $L_{\mathrm{loc}}(k_n)$ of any nth conducting channel is entirely determined by the stratification power spectrum $W(k_x)$, see equation (3.8). Therefore, if $W(2k_n)$ abruptly vanishes for some wave numbers k_n, then $L_{\mathrm{loc}}(k_n)$ diverges and the corresponding propagating mode appears to be fully transparent even for a large length of the waveguide.

Let us take the following binary correlator $\mathcal{W}(x)$ with the corresponding stepwise power spectrum $W(k_x)$:

$$\mathcal{W}(x) = \pi\delta(2k_c x) - \frac{\sin(2k_c x)}{2k_c x}, \tag{3.19a}$$

$$W(k_x) = \frac{\pi}{2k_c}\Theta(|k_x| - 2k_c). \tag{3.19b}$$

Here, $\delta(x)$ is the Dirac delta-function and the characteristic wave number $k_c > 0$ is a correlation parameter to be specified. Applying the method defined by equations (2.13)–(2.15), one can find that the profile of the random stratification having such correlations can be fabricated with the use of the potential

$$V(x) = \frac{V_0}{\sqrt{2\pi k_c}}\left[\pi Z(x) - \int_{-\infty}^{\infty} \mathrm{d}x'\, Z(x - x')\frac{\sin(2k_c x')}{x'}\right]. \tag{3.20}$$

This potential has quite a sophisticated form. It is a superposition of the white noise and the long-range-correlated potential.

For the case under consideration, the inverse value of the mode localization length (3.8) takes the following explicit form:

$$L_{\mathrm{loc}}^{-1}(k_n) = \frac{\pi V_0^2}{16k_c k_n^2}\Theta(k_n - k_c). \tag{3.21}$$

This expression leads to very interesting conclusions.

(i) All *low* propagating modes with the lengthwise wave numbers k_n that exceed the correlation parameter k_c ($k_n > k_c$) have finite mode localization length,

$$L_{\mathrm{loc}}^{-1}(k_n > k_c) = \pi V_0^2 \big/ 16k_c k_n^2. \tag{3.22}$$

For large enough waveguide length $L \gg 16k_c k_1^2 / \pi V_0^2$, all of them are strongly localized. The requirement $k_n > k_c$ implies that the mode indices n of localized channels are restricted from above by the condition

$$n \leqslant N_{\mathrm{loc}} = \left[(kd/\pi)\left(1 - k_c^2/k^2\right)^{1/2}\right]\Theta(k - k_c). \tag{3.23}$$

Therefore, the integer $N_{\rm loc}$ should be regarded as the total number of the localized and non-transparent modes.

(ii) For *high* conducting channels with $k_n < k_c$, the mode localization length diverges,

$$L_{\rm loc}^{-1}(k_n < k_c) = 0. \tag{3.24}$$

Therefore, each of the modes with the index $n > N_{\rm loc}$ has unit partial transmittance $\langle T_n \rangle = 1$ and displays the ballistic transport. Such modes occupy a subset of ballistic and completely transparent channels.

(iii) The value of $N_{\rm loc}$ determines the total number of localized modes. The total number $N_{\rm bal}$ of ballistic modes is evidently equal to $N_{\rm bal} = N_d - N_{\rm loc}$. Since the localized modes do not contribute to the total average waveguide transmittance (3.7), the latter is equal to the number of completely transparent modes $N_{\rm bal}$ and do not depend on the waveguide length L,

$$\langle T \rangle = [kd/\pi] - [\sqrt{(kd/\pi)^2 - (k_c d/\pi)^2}]\Theta(k - k_c). \tag{3.25}$$

We recall that square brackets stand for the integer part of the inner expression. It should be emphasized once more that in contrast to the usual situation related to the hierarchy of mode localization lengths (3.9), now the *low* propagating modes are *localized* and non-transparent, while the *high* modes are *ballistic* with a perfect transparency.

For the correlation parameter $k_c \ll k$, the number of localized modes is of the order of N_d,

$$N_{\rm loc} \approx \left[(kd/\pi)\left(1 - k_c^2/2k^2\right)\right] \qquad \text{for} \quad k_c/k \ll 1. \tag{3.26}$$

Consequently, the number of ballistic modes $N_{\rm bal}$ is small or there are no such modes at all. Otherwise, if $k_c \to k$, the integer $N_{\rm loc}$ turns out to be much less than the total number of propagating modes N_d,

$$N_{\rm loc} \approx [\sqrt{2}(kd/\pi)(1 - k_c/k)^{1/2}] \ll N_d \qquad \text{for} \quad 1 - k_c/k \ll 1. \tag{3.27}$$

Here, the number of transparent modes $N_{\rm loc}$ is large. When $k_c > k_1$, the number $N_{\rm loc}$ vanishes and all modes become fully transparent, in spite of their scattering by random stratification. In this case, the correlated disorder results in a perfect transmission of quantum/classical waves. One can see that the point $k_1 = k_c$ at which the wave number of the first mode k_1 is equal to the correlation wave number k_c, is, in essence, the total mobility edge that separates the region of complete transparency from that where lower modes are localized.

From the above analysis one can conclude that the transmittance (3.25) of a quasi-1D multi-mode structure with a long-range-correlated stratification reveals a quite unexpected *non-monotonic* stepwise dependence on the total wave number k that is controlled by the correlation parameter k_c. An example of this dependence is shown in figure 1.

Within the region where the wave number of the lowest (first) mode k_1 is less than the correlation wave number k_c ($k_1 < k_c$ or, the same, $kd/\pi < \sqrt{(k_c d/\pi)^2 + 1}$), the second term in the expression (3.25) is absent, and all propagating modes are completely transparent. Here, the transmittance exhibits a ballistic *stepwise increase* with an increase of the total wave number k. Each step up arises for an integer value of the mode parameter kd/π when a new conducting channel emerges in the waveguide. Such stepwise increasing behaviour of the total transmittance is similar to that known to occur in quasi-1D ballistic *non-disordered* structures (see, e.g., [45]).

With a subsequent increase of k, the first-mode wave number k_1 becomes larger than k_c ($k_1 \geqslant k_c$ or $kd/\pi \geqslant \sqrt{(k_c d/\pi)^2 + 1}$). Here, the transmittance shows not only standard steps up associated with the first term in equation (3.25) but also *steps down* that are originated

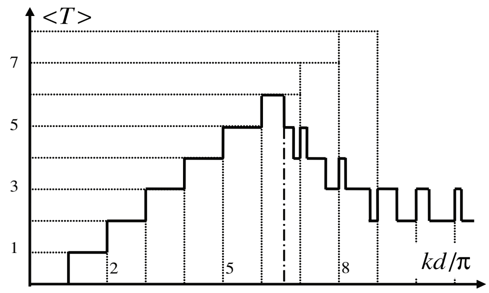

Figure 1. Non-monotonic stepwise dependence of the transmittance (3.25) of stratified structure versus the normalized wave number kd/π for the dimensionless correlation parameter $k_c d/\pi = 6.5$.

from the second term. In contrast to the steps up, the steps down are formed by the correlated scattering and occur due to successive abrupt localization of low modes at the corresponding mobility edges. The first step down occurs at the total mobility edge $k_1 = k_c$, where the first mode is localized. The second step down is due to the mobility edge $k_2 = k_c$ of the second mode, etc. The positions of the steps down are at the integer values of the square root $\sqrt{(kd/\pi)^2 - (k_c d/\pi)^2}$, see equation (3.25), and, in general, do not coincide with the integer values of the mode parameter kd/π. The interplay between steps up and down results in a new kind of stepwise *non-monotonic* dependence of the total quasi-1D transmittance.

These results may find practical applications for the fabrication of electromagnetic/acoustic waveguides, optic fibres and electron nanodevices with a selective transport. For an abrupt dependence of the power spectrum at some energy, the transition from the ballistic to localized transport is expected to be sharp enough in order to observe them experimentally. For example, for the GaAs quasi-1D quantum-well structures with the effective electron mass $m_e = 6.7 \times 10^{-2} m_0$, the Fermi-energy $E_F = 7$ meV and $d = 500$ nm, the number of channels is about 17. Therefore, for the potential with $\pi/k_c \approx 80$ nm, with an increase of k one can observe about 16 local mobility edges characterized by the steps down, after initial six steps up in the conductance dependence on the wave number k.

4. Surface-corrugated guiding systems

In this section, we consider the so-called surface scattering. This type of scattering is due to a disorder that arises inherently in natural atmospheric and oceanic waveguides. Also surface disorder arises either inherently (due to growth defects, fracture, etc) or artificially (e.g., by lithographic preparation) in guiding electron/wave nanodevices.

The commonly used model of the surface scattering is a plane waveguide (or quasi-1D electron wire) of length L and average width d stretched along the x-axis. One (lower) surface of the waveguide is supposed to be of a rough (corrugated) profile $z = \xi(x)$, slightly deviated from its flat average $z = 0$. The other (upper) surface is taken, for simplicity, flat, $z = d$. In other words, the surface-corrugated guiding system occupies the area defined by the relations

$$-L/2 < x < L/2, \qquad \xi(x) \leqslant z \leqslant d. \tag{4.1}$$

The function $\xi(x)$ describing the surface roughness is assumed to be a statistically homogeneous and isotropic random function with the following characteristics:

$$\langle \xi(x) \rangle = 0, \qquad \langle \xi^2(x) \rangle = \sigma^2, \qquad \langle \xi(x)\xi(x') \rangle = \sigma^2 \mathcal{W}(x - x'). \tag{4.2}$$

As in the previous sections, the angular brackets stand for a statistical averaging over the disorder, i.e. in the case of surface disorder over different realizations of the random surface profile $\xi(x)$ with the variance σ^2. The binary correlator $\mathcal{W}(x)$ is normalized to its maximal value, $\mathcal{W}(0) = 1$, and assumed to decrease with increasing $|x|$ on the scale characterizing the mean length of surface corrugation. The roughness power spectrum $W(k_x)$ is defined by equations (2.3*a*)–(2.3*b*) with the normalization (2.4). We also note that the surface corrugations are assumed to be small in height, $\sigma \ll d$, and smooth enough. These limitations are common in the surface-scattering theories that are based on appropriate perturbative approaches [12].

In the x-direction the system is open while in the transverse z-direction the zero Dirichlet boundary conditions are applied to both surfaces, $z = \xi(x)$ and $z = d$. Thus, the analysis of the surface scattering in a quasi-1D guiding system is reduced to the study of the following two-dimensional boundary-value problem:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + k^2\right)\Psi(x, z) = 0, \qquad \Psi(x, z = \xi(x)) = \Psi(x, z = d) = 0. \tag{4.3}$$

Comparing with the models studied in the previous sections, here the Schrödinger (or wave) equation does not contain any scattering potential. In contrast with the bulk scattering, an electron/wave experiences a perturbation due to roughness of the lower surface, i.e. due to the nonuniform boundary condition.

4.1. Single-mode structure

Keeping in mind the relevance of surface scattering of quantum and classical waves to Anderson localization, we, first, consider a single-mode waveguide. In this case, the mode parameter kd/π is restricted by the relation $1 < kd/\pi < 2$ and the number of conducting channels equals to one, $N_d = 1$. The transport through such systems is provided solely by the lowest normal mode with $n = 1$ that propagates with the lengthwise wave number k_1,

$$k_1 = \sqrt{k^2 - (\pi/d)^2}. \tag{4.4}$$

All other waveguide modes with $n \geqslant 2$ are evanescent and do not contribute to the transport properties. From the single-mode condition $1 < kd/\pi < 2$, it follows that the wave number k_1 is confined within the interval

$$0 < k_1 d/\pi < \sqrt{3}. \tag{4.5}$$

Note that the assumed weak surface-scattering condition $\sigma \ll d$ leads to the inequality $k_1\sigma \ll 1$.

As was shown in the papers [24], the transport problem (4.3) for the surface-disordered single-mode waveguide corresponds to that for the 1D-disordered model (2.1) with the lengthwise wave number k_1 in place of k and with the effective surface-scattering potential taken in the form

$$V(x) = \frac{2}{\pi}\left(\frac{\pi}{d}\right)^3 \xi(x). \tag{4.6}$$

Thus, the surface scattering in a single-mode guiding structure should manifest the effect of Anderson localization. In particular, the average transmittance $\langle T(L/L_{\rm loc})\rangle$ is described by the universal expressions (2.6)–(2.11) and, therefore, the transport properties in such single-mode waveguides are governed by the one-parameter scaling. For large localization length, $L_{\rm loc}/L \gg 1$, the system reveals the ballistic transport because its average transmittance $\langle T(L/L_{\rm loc})\rangle \approx 1$, see equation (2.10). In the other limit, $L_{\rm loc}(k_n)/L \ll 1$, the transmittance

$\langle T(L/L_{\mathrm{loc}})\rangle$ is exponentially small in line with equation (2.11). This implies strong electron/wave localization.

Taking into account the specific form (4.6) of the surface-scattering potential and the correlation properties (4.2) for the corrugated surface profile $\xi(x)$, from the general expression (2.12) one can derive the following explicit formula for the surface-scattering localization length [24]:

$$L_{\mathrm{loc}}^{-1}(k_1) = \frac{2\sigma^2}{\pi^2}\left(\frac{\pi}{d}\right)^6 \frac{W(2k_1)}{(2k_1)^2}. \tag{4.7}$$

Since the potential (4.6) is entirely determined by the rough surface profile $\xi(x)$, the surface-scattering localization length (4.7) is specified by the roughness power spectrum $W(k_x)$. Therefore, by a proper fabrication of a random surface profile $\xi(x)$ with specific long-range correlations, one can arrange a desirable anomalous ballistic transport within a given window inside the allowed single-mode region (4.5).

Rough surfaces with prescribed two-point correlations can be constructed with the use of direct generalization of the method discussed in section 2 (for details, see also [46]). Then the resulting surface profile $\xi(x)$ can be obtained due to the following expression:

$$\xi(x) = \sigma \int_{-\infty}^{\infty} \mathrm{d}x'\, Z(x-x')\beta(x'). \tag{4.8}$$

Here, the function $\beta(x)$ is determined by the power spectrum $W(k_x)$ via the relation (2.13) and a delta-correlated random process $Z(x)$ has standard statistical properties (2.15).

Below we demonstrate the possibility of constructing the surface-disordered structures with anomalous ballistic transport by considering two simple examples of a long-range-correlated surface profile.

(a) Let us first consider the waveguide which is non-transparent if the wave number k_1 is less than some value k_c and completely transparent for $k_1 > k_c$. Evidently, such a behaviour can be observed if the transition point (mobility edge) $k_1 = k_c$ is located inside the allowed single-mode interval (4.5),

$$0 < k_c d/\pi < \sqrt{3}. \tag{4.9}$$

In this case, one can get the following expressions for the binary correlator $\mathcal{W}(x)$ and power spectrum $W(k_x)$:

$$\mathcal{W}_1(x) = \frac{\sin(2k_c x)}{2k_c x}, \tag{4.10a}$$

$$W_1(k_x) = \frac{\pi}{2k_c}\Theta(2k_c - |k_x|). \tag{4.10b}$$

With the use of the expressions (4.8), (2.13) and (2.15), the corrugated surface profile with the above correlation properties can be shown to be described by the function

$$\xi_1(x) = \frac{\sigma}{\sqrt{2\pi k_c}} \int_{-\infty}^{\infty} \mathrm{d}x'\, Z(x-x')\frac{\sin(2k_c x')}{x'}. \tag{4.11}$$

Correspondingly, the inverse surface-scattering localization length is of the *step-down* form,

$$L_{\mathrm{loc1}}^{-1}(k_1) = \frac{\sigma^2}{4\pi k_c}\left(\frac{\pi}{d}\right)^6 \frac{\Theta(k_c - k_1)}{k_1^2}. \tag{4.12}$$

Therefore, as the wave number k_1 increases, the localization length $L_{\mathrm{loc1}}(k_1)$ also smoothly increases and then diverges at $k_1 = k_c$. Thus, within the region $k_1 < k_c$ the average transmittance $\langle T(L/L_{\rm loc})\rangle$ is expected to be exponentially small (2.11). The condition for a strong localization to the left from the mobility edge $k_1 = k_c$ reads

$$\frac{L}{L_{\rm loc1}(k_c)} = \frac{\sigma^2 L}{4\pi k_c^3}\left(\frac{\pi}{d}\right)^6 \gg 1. \tag{4.13}$$

In the interval $k_c < k_1 < \pi\sqrt{3}/d$, a ballistic regime occurs with a perfect transparency, $\langle T(L/L_{\rm loc})\rangle = 1$.

(b) The second example refers to a complimentary situation when for $k_1 < k_c$ the waveguide is perfectly transparent and for $k_1 > k_c$ is non-transparent. The corresponding expressions for the correlator $\mathcal{W}(x)$ and its Fourier transform $W(k_x)$ are given by

$$\mathcal{W}_2(x) = \pi\delta(2k_c x) - \frac{\sin(2k_c x)}{2k_c x}, \tag{4.14a}$$

$$W_2(k_x) = \frac{\pi}{2k_c}\Theta(|k_x| - 2k_c). \tag{4.14b}$$

In this case, the corrugated surface is described by a superposition of a white noise and the roughness of the first type,

$$\xi_2(x) = \frac{\sigma}{\sqrt{2\pi k_c}}\left[\pi Z(x) - \int_{-\infty}^{\infty} {\rm d}x'\, Z(x-x')\frac{\sin(2k_c x')}{x'}\right]. \tag{4.15}$$

Correspondingly, the inverse localization length is expressed by the step-up function,

$$L_{\rm loc2}^{-1}(k_1) = \frac{\sigma^2}{4\pi k_c}\left(\frac{\pi}{d}\right)^6 \frac{\Theta(k_1 - k_c)}{k_1^2}. \tag{4.16}$$

As a consequence, in contrast with the first case, here the surface-scattering localization length $L_{\rm loc2}(k_1)$ diverges below the mobility edge $k_1 = k_c$. At this point $L_{\rm loc2}(k_1)$ sharply falls down to the finite value $L_{\rm loc2}(k_c + 0)$ and then smoothly increases with a further increase of k_1. In order to observe the localization regime within the whole region $k_c < k_1 < \pi\sqrt{3}/d$ of finite values of $L_{\rm loc2}(k_1)$, one should assume that a strong localization is retained at upper point $k_1 = \pi\sqrt{3}/d$ of the single-mode region (4.5),

$$\frac{L}{L_{\rm loc2}(\pi\sqrt{3}/d)} = \frac{\sigma^2 L}{12\pi k_c}\left(\frac{\pi}{d}\right)^4 \gg 1. \tag{4.17}$$

Therefore, in this second example the ballistic transport is abruptly replaced by a strong localization at the mobility edge point $k_1 = k_c$.

Below, we demonstrate the above predictions by a direct numerical simulation. For this, the inverse localization length $L_{\rm loc}^{-1}$ was computed with the use of the transfer matrix method. Specifically, the continuous scattering potential (4.6) was approximated by the sum of delta kicks with the spacing δ chosen much smaller than any physical length scale in the model. Thus, the discrete analogue of the 1D wave equation (2.1) was under consideration, with the lengthwise wave number k_1 in place of k and with the surface-scattering potential $V(x)$ in the expression (4.6). In this way, the Schrödinger/wave equation was expressed in the form of a two-dimensional Hamiltonian map describing the dynamics of a classical particle under the external noise determined by the scattering potential $V(x)$. As a result, the analysis of the localization length was reduced to the computation of the Lyapunov exponent $L_{\rm loc}^{-1}$ associated with this map (see details in [1, 2]).

Numerical data reported in figure 2 represent the dependence of the dimensionless Lyapunov exponent $\Lambda = c_0 L_{\rm loc}^{-1}$ on the normalized wave number $K = k_1/k_c$ in the range $0 < K < 2$ corresponding to the single-mode interval. The normalization coefficient c_0 was set

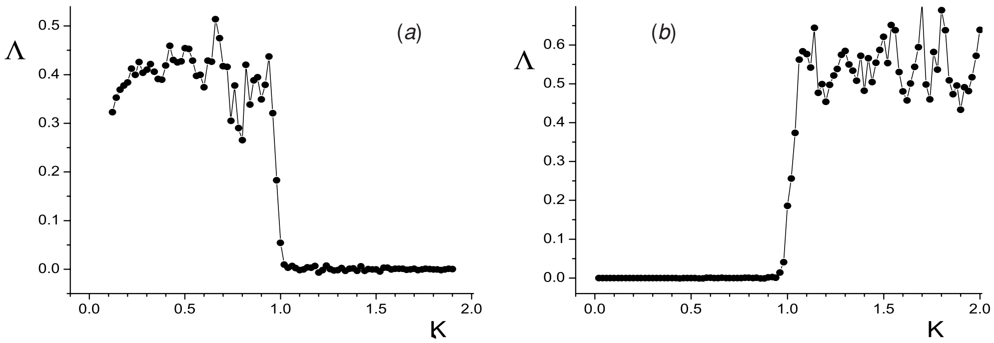

Figure 2. Selective dependence of the rescaled Lyapunov exponent on the wave number for two realizations of a random surface with specific long-range correlations. To show the main effect of the correlations, the complementary dependence of the Lyapunov exponent on K is demonstrated: (a) eigenstates are localized for $K < 1$ and delocalized for $K > 1$ and (b) vice versa, complete delocalization for $K < 1$ alternates with the localization for $K > 1$.

to have $\Lambda = K^{-2}$ for the delta-correlated potential. Two surface profiles $\xi(x)$ were generated according to discrete versions of the expressions (4.11) and (4.15) determining complementary stepwise dependences of the localization length $L_{\mathrm{loc}}(k_1)$ described by equations (4.12) and (4.16).

One can clearly see a non-trivial dependence of Λ on the wave vector K, which is due to specific long-range correlations in $\xi(x)$. The data display sharp dependences of Λ on K when crossing the point $K = 1$. Thus, by taking the size L of the scattering region in accordance with the requirements (4.13) or (4.17), one can arrange anomalous transport in the single-mode guiding structure as is predicted by the analytical theory.

4.2. *Multi-mode waveguide*

Now we examine the correlated surface scattering in *multi-mode* guiding structure with the large total number $N_d > 1$ of conducting channels, see expression (3.6). According to Landauer's concept [18], the *total average transmittance* $\langle T\rangle$ of any quasi-1D conductor can be expressed as a sum of *partial average transmittances* $\langle T_n\rangle$ that describe the transport for every nth propagating normal mode,

$$\langle T\rangle = \sum_{n=1}^{N_d}\langle T_n\rangle. \tag{4.18}$$

From general theory of quasi-1D scattering systems it follows that the transmission properties of any nth conducting channel ($1 \leqslant n \leqslant N_d$) are determined by two *attenuation lengths*, the length $L_n^{(f)}$ of forward scattering and the backscattering length $L_n^{(b)}$. For quasi-1D waveguides with surface disorder, the inverse scattering lengths are given by [12]

$$\frac{1}{L_n^{(f)}} = \sigma^2\frac{(\pi n/d)^2}{k_n d}\sum_{n'=1}^{N_d}\frac{(\pi n'/d)^2}{k_{n'}d}W(k_n - k_{n'}), \tag{4.19}$$

$$\frac{1}{L_n^{(b)}} = \sigma^2\frac{(\pi n/d)^2}{k_n d}\sum_{n'=1}^{N_d}\frac{(\pi n'/d)^2}{k_{n'}d}W(k_n + k_{n'}). \tag{4.20}$$

Here, the lengthwise wave number k_n is defined by the expression (3.5). The results (4.19) and (4.20) can be obtained for the boundary-value problem (4.3) by the diagrammatic Green's function method [12], as well as by the technique developed in [22]. Also, these expressions can be derived by using the invariant imbedding method extended to quasi-1D structures [29]. Note that in a single-mode waveguide with $N_d = 1$, the sum over n' contains only one term with $n' = n = 1$. Therefore, in this case the backscattering length $L_1^{(b)}$ is half the single-mode localization length (4.7), $L_1^{(b)} = L_{\text{loc}}(k_1)/2$.

The expressions (4.19) and (4.20) manifest that, in general, both attenuation lengths are contributed by scattering of a given nth propagating mode into all other modes. This is the case when, for example, a rough surface profile is either delta-correlated random process ('white noise') with constant power spectrum, $W(k_x) = const$, or has fast decreasing binary correlator $\mathcal{W}(x)$ (or, the same, slowly decreasing roughness power spectrum $W(k_x)$). Therefore, the quasi-1D systems reveal three typical transport regimes, the regimes of a *ballistic*, *diffusive* (metallic) and *localized transport*.

Another peculiarity is that the expressions (4.19) and (4.20) display rather strong dependence on the mode index n, namely, the larger the number n the smaller the corresponding mode scattering lengths and as a consequence, the stronger is the scattering of this mode into the others. As a result, there is the following hierarchy of mode scattering lengths:

$$L_{N_d}^{(f,b)} < L_{N_d-1}^{(f,b)} < \cdots < L_2^{(f,b)} < L_1^{(f,b)}. \tag{4.21}$$

The smallest mode attenuation lengths, $L_{N_d}^{(f)}$ and $L_{N_d}^{(b)}$, belong to the highest (last) channel with the mode index $n = N_d$, while the largest scattering lengths, $L_1^{(f)}$ and $L_1^{(b)}$, corresponds to the lowest (first) channel with $n = 1$. As was shown in [29], due to this hierarchy of scattering lengths, even in the absence of correlations in the corrugated surface $\xi(x)$, a very important phenomenon of the *coexistence* of ballistic, diffusive and localized transport arises. Specifically, while lowest modes can be in the ballistic regime, the intermediate and highest modes can exhibit the diffusive and localized behaviour, respectively. This effect seems to be generic for the transport through the waveguides with random surfaces.

One can see now that unlike the single-mode case, the concept of one-parameter scaling is no more valid for the transport in multi-mode systems. There are two points that should be stressed in this respect. On the one hand, the average partial transmittances $\langle T_n \rangle$ entering equation (4.18) are very different for different conducting channels. On the other hand, and what is even more important, all propagating modes turn out to be mixed due to the inter-mode transitions. Therefore, the transmittance $\langle T_n \rangle$ of a given nth mode depends on the scattering into all modes.

From this analysis one can conclude that for multi-mode structures with surface disorder the role of specific long-range correlations should be much more sophisticated in comparison with that discussed above for single-mode waveguides. First, such correlations should result in the suppression of the interaction between different propagating modes. This non-trivial fact turns out to be crucial for the reduction of a system of mixed channels with quasi-1D transport, to the coset of independent waveguide modes with a purely 1D transport. Second, the same correlations can provide a complete transparency of each independent channel, similar to what happens in strictly 1D geometry.

To demonstrate these effects, let us take a random surface profile $\xi(x)$ of the form

$$\xi(x) = \frac{\sigma}{\sqrt{\pi k_c}} \int_{-\infty}^{\infty} \mathrm{d}x'\, Z(x - x') \frac{\sin(k_c x')}{x'}, \tag{4.22}$$

with the slowly decaying (in average) binary correlator which results in the 'window' function for the roughness power spectrum,

$$\mathcal{W}(x) = \frac{\sin(k_c x)}{k_c x}, \tag{4.23a}$$

$$W(k_x) = \frac{\pi}{k_c}\Theta(k_c - |k_x|), \qquad k_c > 0. \tag{4.23b}$$

From equations (4.19) and (4.20) one can see that in the case of long-range correlations in a disordered surface (4.22), the number of modes into which a given nth mode is scattered, i.e. the actual number of summands in equations (4.19) and (4.20), is entirely determined by the width k_c of the rectangular power spectrum (4.23*b*). It is clear that if the distance $|k_n - k_{n\pm1}|$ between neighbouring quantum values of k_n is larger than the correlation width k_c,

$$|k_n - k_{n\pm1}| > k_c, \tag{4.24}$$

then the transitions between all propagating modes are forbidden. As a consequence, the sum over n' in the expression (4.19) for the inverse forward-scattering length contains only diagonal term with $n' = n$ which describes direct intra-mode scattering *inside* the channels. Moreover, each term in the sum of equation (4.20) for the inverse backscattering length is equal to zero. As a result, the following interesting properties arise.

(i) All *high* propagating modes with indices n that satisfy the condition (4.24) turn out to be independent of the others in spite of the interaction with rough surface. Therefore, they form a coset of 1D non-interacting conducting channels with finite length of forward scattering $L_n^{(f)}$ and infinite backscattering length $L_n^{(b)}$,

$$\frac{1}{L_n^{(f)}} = \frac{\pi\sigma^2}{k_c}\frac{(\pi n/d)^4}{(k_n d)^2}, \qquad \frac{1}{L_n^{(b)}} = 0. \tag{4.25}$$

(ii) As is well known from the standard theory of 1D localization (see, e.g., [15, 16, 26]), the transport through any 1D-disordered system is determined only by the backscattering length $L_n^{(b)}$ and does not depend on the forward-scattering length $L_n^{(f)}$. Since the former diverges for every independent channel in line with the expression (4.25), all of them exhibit the ballistic transport with the partial average transmittance $\langle T_n\rangle = 1$. This means that according to the Landauer's formula (4.18), the transmittance of the coset of such independent ballistic modes is simply equal to their total number.

(iii) As for *low* propagating modes with the indices n contradicting to the condition (4.24), they remain to be mixed by surface scattering because the roughness power spectrum (4.23*b*) is non-zero for them, $W(k_n - k_{n'}) = \pi/k_c$. These *mixed modes* have finite forward and backscattering lengths. Therefore, for large enough waveguide length L, they do not contribute to the total transmittance $\langle T\rangle$ and the latter is equal to the number of independent ballistic modes.

Note that the distance $|k_n - k_{n\pm1}|$ between neighbouring quantum wave numbers k_n and $k_{n\pm1}$ increases as the mode index n increases. Therefore, the inequality (4.24) restricts the mode index n from below. That is why, in contrast with the conventional situation associated with the hierarchy of mode scattering lengths (4.21), the low modes are mixed and non-transparent, while high propagating modes are independent and ballistic. Because of the sharp behaviour of the roughness power spectrum (4.23*b*), the transition from mixed to independent modes is also sharp.

More analytical results can be obtained for waveguides with large number of conducting channels (3.13), if the quantum numbers n of independent ballistic modes are also large,

$N_d \approx kd/\pi \geqslant n \gg 1$. In this case, the inequality (4.24) is reduced to the requirement $|\partial k_n/\partial n| > k_c$ which can be rewritten in the following explicit form:

$$n > N_{\text{mix}} = \left[\frac{(kd/\pi)}{\sqrt{1 + (k_c d/\pi)^{-2}}}\right]. \tag{4.26}$$

We recall that square brackets stand for the integer part of the inner expression.

The condition (4.26) determines the total number N_{mix} of mixed non-transparent modes, the total number $N_{\text{bal}} = N_d - N_{\text{mix}}$ of independent ballistic modes and the critical value of the mode index n that divides these two groups. All propagating modes with $n > N_{\text{mix}}$ are independent and fully transparent, otherwise, they are mixed for $n \leqslant N_{\text{mix}}$ and characterized by finite scattering lengths $L_n^{(f)}$ and $L_n^{(b)}$. Therefore, the total average transmittance (4.18) of the multi-mode structure is

$$\langle T\rangle = [kd/\pi] - [(kd/\pi)/\alpha_c], \qquad \alpha_c = \sqrt{1 + (k_c d/\pi)^{-2}}. \tag{4.27}$$

The numbers N_{mix} and N_{bal} of mixed non-transparent and independent ballistic modes are governed by two parameters, the mode parameter kd/π and the dimensionless correlation parameter $k_c d/\pi$. In the case of 'weak' correlations when $k_c d/\pi \gg 1$, the number of mixed modes N_{mix} is of the order of N_d,

$$N_{\text{mix}} \approx \left[\left(\frac{kd}{\pi}\right) - \frac{1}{2}\left(\frac{kd}{\pi}\right)\left(\frac{k_c d}{\pi}\right)^{-2}\right] \qquad \text{for} \quad k_c d/\pi \gg 1. \tag{4.28}$$

Consequently, in this case the number of ballistic modes N_{bal} is small or there are no such modes at all. If the parameter $k_c d/\pi$ tends to infinity, $k_c d/\pi \to \infty$, the rough surface profile becomes white-noise-like and, naturally, $N_{\text{mix}} \to N_d$.

The most appropriate case is when a random surface profile is strongly correlated so that the correlation parameter is small, $k_c d/\pi \ll 1$. Then the number of mixed non-transparent modes N_{mix} is much less than the total number of propagating modes N_d,

$$N_{\text{mix}} \approx \left[\left(\frac{kd}{\pi}\right)\left(\frac{k_c d}{\pi}\right)\right] \ll N_d \qquad \text{for} \quad k_c d/\pi \ll 1. \tag{4.29}$$

Therefore, the number of independent modes N_{bal} is large. When the correlation parameter $k_c d/\pi$ decreases and becomes anomalously small, $k_c d/\pi < (kd/\pi)^{-1} \ll 1$, the number of mixed modes N_{mix} vanishes and *all modes* become independent and perfectly transparent. Evidently, if the correlation parameter $k_c d/\pi$ vanishes, $k_c d/\pi \to 0$, the roughness power spectrum (4.23*b*) becomes delta-function-like and, as a consequence, $N_{\text{mix}} = 0$. In this case, the correlated disorder results in a perfect transmission of electrons or waves.

Finally, let us briefly discuss the expression (4.27) for the total average transmittance. In figure 3, an unusual stepwise dependence of $\langle T\rangle$ on the mode parameter kd/π is shown that is governed by the width k_c of the rectangular power spectrum (4.23*b*).

Let us discuss this figure. Within the region $kd/\pi < \alpha_c$, the inter-mode transitions caused by the specific surface correlations are forbidden for all conducting channels. Therefore, all propagating modes are independent and ballistic, and the second term (the number of mixed modes) in the expression (4.27) for the total transmittance is equal to zero. Here, the transmittance exhibits a ballistic stepwise increase with an increase of the parameter kd/π. Each step up arises for an integer value of the mode parameter kd/π when a new conducting channel emerges in the guiding structure.

Otherwise, when $kd/\pi \geqslant \alpha_c$, in addition to the standard steps up originated from the first term in equation (4.27), there are also the steps down connected with the second term. These steps down are provided by the correlated surface scattering and arise when a successive low

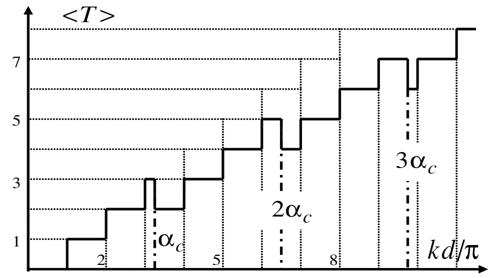

Figure 3. Stepwise transmittance (4.27) of surface-disordered guiding system versus the mode parameter kd/π. The value of the normalized correlation parameter $k_c d/\pi = 0.32$.

mode becomes mixed and non-transparent. In other words, the positions of the nth steps down are at the mobility edge point $kd/\pi = n\alpha_c$ where the nth conducting channels closes. Since the values of the ratio $kd/\pi\alpha_c$ are determined by the correlation parameter k_c, the positions of steps down, in general, do not coincide with those of steps up. The situation may also occur when the steps up and down cancel each other within some interval of the mode parameter kd/π. The experimental observation of the discussed non-conventional dependence seems to be highly interesting.

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